AD-A182 456 OPTIMAL MAINTENANCE AND INFERENCE IN RELIABILITY(U) 1/1
COLUMBIA UNIV NEW YORK DEPT OF STATISTICS
M N KATEHAKIS ET AL 01 JUN 84 AFOSR-TR-0898
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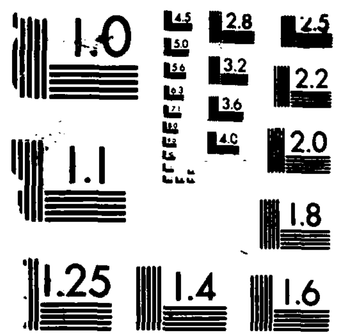

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REPORT DOCUMENTATION PAGE

AD-A182 456

1b. RESTRICTIVE MARKINGS	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited
4. PERFORMING ORGANIZATION REPORT NUMBER(S) Columbia University	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 87-0898
6a. NAME OF PERFORMING ORGANIZATION / 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION AFOSR/NM
6c. ADDRESS (City, State and ZIP Code) Dept. of Statistics New York, N.Y. 10027	7b. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB DC 20330-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR / 8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-84-0136
8c. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB DC 20332-6448	10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
61102F	2304	A5	

11. TITLE (Include Security Classification)
Optimal Maintenance and Inference in Reliability

12. PERSONAL AUTHOR(S)
H. Robbins

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Final	FROM 6/1/84 TO 5/31/86	6/1/84	60

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The main results obtained were in the following areas: 1. sequential allocation problems, 2. Optimal Maintenance policies, 3. empirical Bayes and prediction, and 4. nonparametric ~~line~~ Bayes estimation and documenting work under this grant.

DTIC ELECTE JUL 2 1 1987 S D D

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
Maj. Brian W. Woodruff	(202)767-5027	NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

SECURITY CLASSIFICATION OF THIS PAGE

AFOSR-TR- 87-0898

Progress Report on Research on

Optimal Maintenance and Inference In Reliability

by

Michael N. Katehakis[1], Lynn Kuo[1] and, H. Robbins[2]

The main results obtained during the period 5/84 to 6/86 are on the following:

I) **Sequential Allocation Problems.**

We considered the general, discrete time, effort allocation problem known as the Multi-Armed Bandit problem . This class of problems was first formulated by Robbins (1952), and it is an important sequential control problem with a tractable solution. A simple version of it can be stated as follows. There are N independent projects (e.g., statistical populations, manufacturing machines, maintenance actions, etc.). The state of the i-th project at time t is denoted by $x_i(t)$ and it belongs in a set of states S_i (which in the simplest case is a countable set). At each point of time $t = 0,1,\ldots$ one can work on one project only and if the i-th of them is selected receives a reward $R(t) = r_i(x_i(t))$ and its state changes according to a known Markovian transition rule $P_i(x_i(t))$ (i.e., the probabilities $P(x_i(t+1) = y \mid x_i(t) = x)$ are known) while the states of all other projects remain unchanged. The states of all projects are observable and the objective is to determine a dynamic effort allocation rule π so as to minimize the expected total discounted reward $E_\pi(\sum_{t=0}^{\infty} \beta^t R(t) \mid x(0))$, for some discount factor β in $(0,1)$.

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Gittins and Jones (1974) (c.f. Gittins (1979), Whittle (1980)) showed that this general problem can be reduced to N one dimensional problems. Each of the latter problems involves a single project, and its solution is the dynamic allocation index value for the current state of the project. At each point of time an optimal policy for the original problem is such that it allocates effort to the project with the largest index value in the then current state. If the present state of a project is x then the corresponding value of the index is given by one of the following, equivalent, expressions:

$$(1)\quad m(x) = \sup_{\tau}\{[E(\textstyle\sum_{t=0}^{\tau-1} \beta^{t}R(t) \mid x(0))]/[1 - E(\beta^{\tau})] \ ,$$

$$(2)\quad m(x) = \inf\{M : \sup_{\tau}\{E(\textstyle\sum_{t=0}^{\tau-1} \beta^{t}R(t) + M\beta^{\tau} \mid x(0))\} = M \} \ ,$$

where, in (1) , and (2), τ is a stopping time associated with the process that describes the evolution of the state of the project under consideration.

It is a difficult task to compute the indices via relations (1) and (2).

Subsequently, Katehakis and Veinott (1985) obtained the following characterization for the index:

$$(3)\quad m(x) = \sup_{\sigma}\{[E(\textstyle\sum_{t=0}^{\infty} \beta^{t}R(t) \mid x(0))] \ .$$

where, in (3) σ , is a return time to state $x(0)$. In (3) $m(x)$ is the value of the problem in which at any point in time we have to decide whether to continue using the project or to return it to its initial state (at time σ) and start all over again .

Characterisation (3) reduces the problem of computing $m(x)$ into a standard and easy problem of dynamic programming.

Also, in Katehakis and Veinott (1985) a simpler proof of the original theorem of Gittins and Jones (and that of Whittle) was given.

In Katehakis and Derman (1985c) the characterisation (3) is used to compute optimal policies in the context of a sequential clinical trials problem.

II) Optimal Maintenance Policies.

We consider a system of known structure that is composed of N components and is maintained by R repairmen, where R is less than N. Component functioning and repair times are random variables with known distributions. The problem is to characterize dynamic maintenance policies; i.e., rules for choosing to which failed components repairmen are assigned, that yield a maximum value to a system measure of performance such as the expected discounted system operation time and the average expected system operation time.

Under appropriate assumptions, at any time the status of all components is given by a vector $\underline{x} = (x_1,\dots,x_N)$ with $x_i = 1$ or 0 if the i-th component is functioning or failed. Similarly, the state of the system is given by the structure function ϕ, where $\phi(\underline{x}) = 1$ or 0 if the system is functioning or failed when component status is $\underline{x}$.

In Katehakis and Derman (1985a) (see also, Katehakis (1980)) we considered systems that are composed of highly reliable components. We extended work done in Smith (1978) by providing a formulation of the general problem along the lines of Markovian Decision Theory. Systems composed of highly reliable components are modeled by assuming that the failure rate for the i-th component is of the form $\rho\mu_i$, $1 \le i \le N$, for some scalar $\rho > 0$. Thus, for small values of ρ all components are highly reliable. Asymptotic power series expansions of the expected discounted nonfunctioning time $D_\pi(\underline{x},\beta)$ are obtained; i.e.,

$$D_\pi(\underline{x},\beta) = \sum_{k=0}^{\infty} \rho^k D_\pi^{(k)}(\underline{x},\beta) \quad ,$$

where β denotes the discount rate. For small values of ρ optimal policies were determined by minimizing the leading coefficients of the above power series. It was shown that there exists an interval $(0,\rho^*)$ with the property that if it contains the failure rates of all components, then the asymptotically optimal policy under consideration is optimal. Recursive formulas for computing the coefficients $D_{\pi}^{(k)}(\underline{x},\beta)$ were obtained and were used to derive partial characterizations of asymptotically optimal policies.

Finally, the explicit form of asypmptotically optimal policies for systems of specific structure such as for the series-parallel (for $R \geq 2$) and a system composed of parallel subsystems connected in series (for $R = 1$) were given.

III) Empirical Bayes and Prediction.

Let $f(\cdot|\theta)$ be a given parametric family of probability density functions with respect to some σ-finite measure μ such that

$$\int x f(x|\theta)d\mu(x) = \theta \quad \text{for all } \theta . \tag{4}$$

Let (θ,X,Y), (θ_i,X_i,Y_i), $i=1,2,\ldots$, be i.i.d. random vectors such that θ has some (unknown) distribution function G , while conditionally on θ , X and Y are independent with respective probability density functions $f(x|\theta)$ and $f(y|\lambda\theta)$, where λ is some constant. Finally, let $u(x)$ be a given function dictated by practical considerations.

Let us consider three problems:

<u>Problem I.</u> Estimate the random quantity

$$S_n = \Sigma_1^n u(X_i)\theta_i$$

by some function of $X_1,\ldots,X_n$.

<u>Problem I.</u> Predict the random quantity

$$S_n' = \Sigma_1^n u(X_i)Y_i$$

by some function of $X_1,\ldots,X_n$ and λ , when λ is known (e.g., $\lambda = 1$).

<u>Problem III.</u> When λ is unknown estimate it by some function of $X_1,\dots,X_n$ and $Y_1^*,\dots,Y_n^*$, where $Y_i^* = u(X_i)Y_i$.

One way of solving these problems is to try to find a function $v(x)$ such that

$$\int v(x)f(x|\theta)d\mu(x) = \theta \int u(x)f(x|\theta)d\mu(x) \quad \text{for all } \theta , \tag{5}$$

and then to define

$$V_n = \Sigma_1^n v(X_i) . \tag{6}$$

From (5) it follows that irrespective of the the nature of the distribution function G of θ,

$$ES_n = EV_n , \quad ES_n' = \lambda EV_n . \tag{7}$$

so that S_n can be estimated by V_n, S_n' by λV_n when λ is known, and unkown λ can be estimated by S_n'/V_n. The asymptotic distributions of these estimators have been obtained, and asymptotic confidence intervals for S_n, S_n', and λ have been found.

We have found solutions of the basic equation (2) for many of the most common parametric families, and have established the optimality of the corresponding "u,v" estimators in some cases. The practical importance of these results is indicated in the proposal for future work.

IV) **Nonparametric Linear Bayes Estimation in Quantal Bioassay.**

An experimenter intends to test the strength of a material by applying shocks at different levels to groups of components. The response to a shock is assumed to be dichotomous: either damaged or not. We observe that $k = (k_1,\dots,k_L)$ components are damaged when $n = (n_1,\dots,n_L)$ components are tested at stress levels $t = (t_1,\dots,t_L)$ respectively $(t_1 < t_2 < \dots < t_L)$. The tolerance distribution is defined by $F(t) :=$ probability of damage at stress level t.

In Kuo L. (1986) a nonparametric linear Bayes estimator for F is developed, where the prior is assumed to be Ferguson' Dirichlet process (1973); we have studied its asymptotic properties and have given numerical comparisons for some cases.

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